# **Statistics Questions**

## **1. Definitions**

1. Define the terms “sample space”, “event” and “random variable” and give an example of each. [10 points]
2. What is an indicator random variable and what is the probability mass function of a discrete random variable? [5 points]
3. Define the conditional probability of an event and state Bayes Theorem. [5 points]
4. Explain what is meant by “marginalization”. [5 points]

## **2. Basic Questions**

1. Suppose a string of n bits is sent across a lossy link. In how many ways can 2 bit errors occur? When n=3, list the possible set of bit error patterns. [5 marks]
2. In how many ways can the letters MISSISSIPPI be rearranged? [10 marks]
3. In poker a straight flush consists of 5 cards in order, all of the same suit e.g. the 2,3,4,5,6 of clubs. How many straight flushes are possible? [10 marks]
4. Six people get into an elevator at the ground floor of a hotel which has 10 upper floors. Assuming each person gets off at a floor uniformly at random, what is the probability that no two people get off at the same floor? [10 marks]
5. A random variable X has P(X=x)=x/15 for x=1, 2,3,4,5, and P(X=x) is 0 for all other values. What is the mean and variance of X? [5 marks]
6. You perform the following experiment: you take a six-sided die, and roll it. If the number that comes up is six, you stop; otherwise you repeat.
7. What is the distribution of the number of times you roll the die? [5 marks]
8. What is the expected number of rolls? [10 marks]

## **3. Using Bayes Rule**

1. Suppose we have two bags, labeled A and B.
2. Bag A contains 3 white balls and 1 black ball, bag B contains 1 white ball and 3 black balls.
3. We toss a fair coin and select bag A if it comes up heads and otherwise bag B. From the selected bag we now draw 5 balls, one after another, replacing each ball in the bag after it has been selected (the bag always contains 4 balls each time a ball is drawn).
4. We observe 4 white balls and 1 black ball.
5. What is the probability that we selected bag A ? *(Hint: use Bayes Rule.)* [20 points]
6. State Bayes Rule. [5 points]
7. Suppose 1% of computers are infected with a virus. There is an imperfect test for detecting the virus.
8. When applied to a computer with the virus the test gives a positive result 90% of the time.
9. When applied to a computer which does not have the virus, the test gives a negative result 99% of the time.
10. Suppose that the test is positive for a computer. What is the probability that the computer has the virus? [10 points]

## **4. Expectations & Random Variables**

1. Define the expected value of a random variable. Give a proof that the expected value is linear i.e. E[X+Y]=E[X]+E[Y] for random variables X and Y. [5 points]
2. Define what it means for two random variables to be independent. Give a proof that when two random variables X and Y are independent then E[XY]=E[X]E[Y]. [5 points]
3. Define the covariance and correlation of two random variables X and Y. [ 5 points]
4. You invent a game where the player bets €1, and rolls two dice. If the sum is 7, the player wins €k, and otherwise loses their €1 bet.
5. Define the expectation and variance of a discrete random variable. [5 points]
6. What is the expected reward in this game? [5 points]
7. What value of k makes the game fair (i.e. makes the expected reward zero)? What is the variance of the reward in this case? [10 points]
8. For two independent random variables X and Y show that Var(X+Y)=Var(X)+Var(Y). Hint: Recall that E[X+Y]=E[X]+E[Y] and that when X and Y are independent then E[XY]=E[X]E[Y] [10 points]
9. Suppose that you play the game 2 times in a row with k=5. What is the expected value of the reward (i.e. of the aggregate winnings after playing 2 times)? What is its variance ? What is the expectation and variance of the reward after 100 plays? [5 points]

## **5. Dunno**

1. A bag contains 30 balls, of which 10 are red and the other 20 blue.
2. Suppose you take out 8 balls from this bag, with replacement.
3. What is the probability that among the 8 balls in this sample exactly 3 are red and 5 are blue? [5 points]
4. Now suppose that the balls are taken out of the bag without replacement.
5. What is the probability that out of 8 balls exactly 3 are red and 5 are blue? [10 points]
6. Suppose we roll a red die and a green die:
7. What is the sample space for this experiment? [5 points]
8. What is the probability that the number on the green die is larger than the number on the red die? [5 points]
9. Define what it means for two events E and F to be independent. [5 points]
10. Let event E be that the sum of the dice equals 2 or 3 and event F be that the sum equals 3. Are E and F independent? Explain with reference to the definition given above. [10 points]